S282 Astronomy

CMB power spectra

Study time: 60 minutes

Summary

In this activity you will use the *Cosmological modeller* software package to predict the power spectrum of the cosmic microwave background (CMB) in a range of cosmological models. This activity will help you appreciate how the values of cosmological parameters can be constrained by measurements of the CMB.

You should have read to the end of Chapter 7 of *An Introduction to Galaxies and Cosmology* before starting this activity.

Learning outcomes

- Understand the concept of mathematical modelling.
- Understand the process of testing a model by comparing predictions of the model with observed results.

Background to the activity

In Section 7.5 of *An Introduction to Galaxies and Cosmology* you saw how intrinsic anisotropies in the cosmic microwave background could be described mathematically in terms of a power spectrum and how this allows a more detailed analysis to be made than by simple visual assessment of anisotropy maps. Specifically, the mathematical description provided by the power spectrum allows the results from observational measurements to be compared *quantitatively* with the predictions of different theoretical models.

The Power spectrum calculator section of the *Cosmological modeller* program used in this activity uses the FRW models, as described in Chapter 7, to simulate the fluctuations in the cosmic microwave background and compare the resulting predicted angular power spectrum with the power spectrum of the WMAP anisotropy map.

In this activity you will explore the sensitivity of the power spectrum to a number of cosmological parameters such as the density of matter and the cosmological constant. In doing so you will get a feel for the way that the most probable values of these parameters can be determined by comparing the power spectra predicted by the models with real observational data such as that from WMAP.

The activity

The *Cosmological modeller* program has a section that predicts the expected variations in the cosmic microwave background for different cosmological models. It does this by calculating and plotting the angular power spectrum of the CMB as described in Section 7.5 of *An Introduction to Galaxies and Cosmology*. Although a brief description of this plot is given here, you may wish to refer to Sections 7.5.2 and 7.5.3 of *An Introduction to Galaxies and Cosmology* whilst working through this activity.

- Start the S282 Multimedia guide, and open the folder called 'Cosmology', then click on the icon for CMB power spectra.
- Click Start to open the *Cosmological modeller* program.
- Click on the CMB power spectra button to open the power spectrum section of the program.

Calculating the power spectrum

The Power spectrum calculator section of the program allows to you explore the effects of changing the various parameters that specify the model and hence compare the predictions of different cosmological models.

The power spectrum calculator has a data entry area where you can enter the parameters that specify the model (Figure 1).

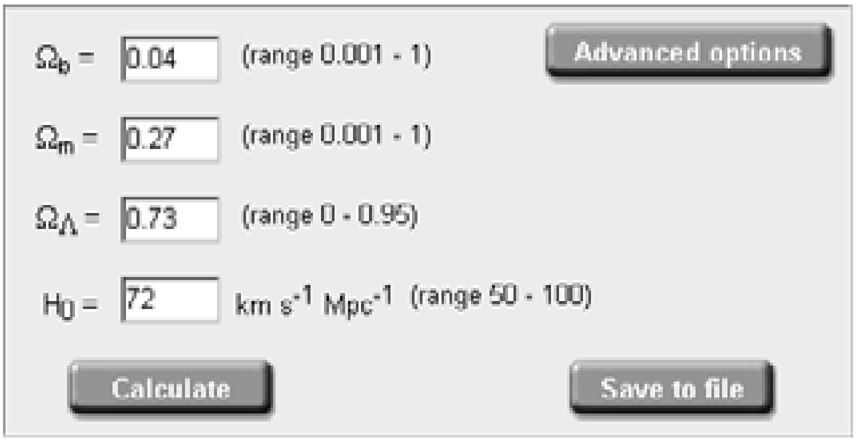


Figure 1 The power spectrum calculator.

Note that the density parameters that the program works with are all the present-day values of these parameters. So strictly speaking, these parameters should all be written in the form that explicitly shows that they relate to the present time, such as $\Omega_b(t_0)$ or $\Omega_{b,0}$. However for brevity the program (and these notes) display the density parameters without the t_0 or the subscript 0. However, as you work thought this activity you should remember that it is the present-day values of these parameters that are referred to.

When the program starts, the model parameters are set to the default values shown in Figure 1. Notice that the input to the program includes the parameter that quantifies the density of baryonic matter Ω_b. Note also that Ω_m represents the total matter density – baryonic plus dark matter – and therefore includes Ω_b. In this model baryonic matter contributes about 15% of the total matter density.

- To start with, leave the parameters set to the values shown above and click on the Calculate button to generate the power spectrum for this model.

The calculation may take a few minutes. Once the calculation is complete, you should obtain a curve similar to that shown in Figure 7.22 of *An Introduction to Galaxies and Cosmology*, with peaks at values of l of about 200, 500 and 800.

When interpreting these power spectra remember that the horizontal axis represents the angular scale of the fluctuations in the CMB (with smaller angular scales to the right), and the vertical scale represents the angular power. The angular power provides a quantitative measure of the size of fluctuations at a given angular scale.

Question 1

The scale on the horizontal axis of this plot runs from angular scales of a few degrees down to about 0.1 degree. Compare the angular scales that correspond to the peaks in this power spectrum with the resolution of COBE all-sky map (*An Introduction to Galaxies and Cosmology*, Figure 7.16) and that of the WMAP all-sky map (*An Introduction to Galaxies and Cosmology*, Figure 7.21). Which of these maps (COBE or WMAP) do you think could show fluctuations that correspond to the peaks is this power spectrum?

In answering Question 1 you should have reached the conclusion that the displayed power spectrum shows variation over scales that were mapped by the WMAP mission. In fact, the default values of the program are close to the values given in Section 7.5.4 of *An Introduction to Galaxies and Cosmology* as the best fit to the WMAP data. You can display the WMAP data and best-fit model to these data by selecting the options (in tick boxes at the upper right-hand part of the screen) MAP data and best fit to data.

In the next section you will compare this prediction with that of a very different model. At any stage through these exercises, you can compare the predictions of a given model with the WMAP data by using the MAP data and best fit to data options.

Comparison with critical model

Prior to the Type Ia supernova measurements described in Section 7.3.2 of *An Introduction to Galaxies and Cosmology*, one of the preferred cosmological models was the matter-dominated critical density model with $\Omega_m = 1.0$ and $\Omega_\Lambda = 0.0$. In this section of the activity you will look at what this model predicts for the fluctuations in the CMB.

- For the critical Universe model the density parameters are set to $\Omega_m = 1.0$ and $\Omega_\Lambda = 0.0$. If the proportion of baryonic matter is to remain at 15%, what would be the value of Ω_b required for this model?

- ❑ Since the total matter density Ω_m is 1.0, in order to maintain the 15% proportion of baryonic matter, Ω_b should be set to 0.15.

■ Enter the values $\Omega_m = 1.0$ and $\Omega_\Lambda = 0.0$ together with the appropriate value for Ω_b (= 0.15), and select Calculate again. After some delay, the curve for the critical model should be displayed, superimposed on the previous curve.

Question 2

Looking at the differences between the two curves, describe any similarities and differences between them. How would the corresponding all-sky maps of the CMB differ from one another?

The effect of varying the cosmological constant

Clearly the comparison with the critical density model represents quite a drastic change which would give quite a different pattern to the CMB. We will now consider some more subtle differences between models. First, let's investigate the effect of changing the amount of cosmological constant by varying Ω_Λ.

Because there are a number of parameters that can be varied, it would be very easy to get confused by changing several properties of the model at once. In order to make meaningful comparisons it is necessary to work within certain constraints, so that only one characteristic of the model is changed at any one time. Specifically, to isolate the effect of varying Ω_Λ we want to maintain a flat Universe and to keep the proportion of baryonic matter constant (you will investigate the effect of changing this proportion in the next section).

■ As the value of Ω_Λ is varied, what effects do these constraints have on the values of Ω_m and Ω_b?

❑ In order to maintain a flat Universe, $(\Omega_m + \Omega_\Lambda)$ should add up to 1.0. Thus, if Ω_Λ is changed, Ω_m must also be changed to maintain a total of 1.0.

To keep the proportion of baryonic matter constant Ω_b should be changed at the same time to keep its value at 15% of the value of Ω_m.

■ Based on these considerations, select Clear graph and make three plots with the following combinations of parameters, leaving H_0 set to 72 km s^{-1} Mpc^{-1} throughout (don't clear the graph between plots – just enter the parameters and select Calculate, so that you end up with the three curves superimposed).

Model	Ω_b	Ω_m	Ω_Λ
1	0.06	0.4	0.6
2	0.04	0.27	0.73
3	0.03	0.2	0.8

These three sets of parameters represent a small variation of Ω_Λ either side of the best-fit value, keeping the proportion of baryonic matter fixed at 15%.

■ How much difference is there between these three plots, compared to the critical model plots? How easy is it to distinguish between these three models?

❑ There is very little difference between these three models. Varying the cosmological constant within the constraints given above appears to have had little effect on the prediction for the CMB.

You may wish to print a copy of these plots before proceeding. (Select the Print option from the File menu.)

So varying the parameters in a systematic way has allowed us to isolate the effects of a single property of the model. In this case, varying the proportion of cosmological constant by a small amount either way has relatively little effect on the predicted power spectrum, although, as you saw earlier, eliminating it entirely did produce a large effect. This tells us that a cosmological constant is definitely required in the model. Once Ω_Λ is added, the power spectrum is relatively unchanging and is not very sensitive to small changes either side of the 'best-fit' value of 73%. We will consider implications of this lack of sensitivity to the exact value of Ω_Λ in the conclusion to this activity.

The effect of varying the proportion of baryonic matter

In a similar vein to the previous section, let's now explore the effect of changing the proportion of baryonic matter by varying Ω_b slightly whilst keeping the other parameters fixed.

- Select Clear graph and again make three plots, this time using the following parameter combinations.

Model	Ω_b	Ω_m	Ω_Λ
4	0.02	0.27	0.73
5	0.04	0.27	0.73
6	0.06	0.27	0.73

Question 3

Compare these three plots with the three plots made above when you investigated the effect of varying Ω_Λ. Which parameter (Ω_Λ or Ω_b) has the larger effect on the shape of the power spectrum, and why?

You may want to explore further using different combinations of parameters. Again, try to make changes subject to reasonable constraints and work to a plan in order to generate meaningful comparisons.

Conclusions

During the course of this activity you should have developed a feel for the way in which the power spectrum allows detailed mathematical comparisons of different models to be made. By making small adjustments to the parameters the predictions of the model can be 'fine-tuned' to obtain a best fit to the observational data. In doing so it has been possible to come up with a combination of the parameters Ω_Λ, Ω_m and Ω_b that gives a good agreement with the power spectrum of the WMAP anisotropy map.

The results of such curve-fitting activities should be taken more as an indication of the *probable values* of these parameters rather than as conclusive proof. For one thing, you have seen that the model is more sensitive to the proportion of

baryonic matter than to the exact value of cosmological constant, so is a better predictor of some parameters than of others. Furthermore, it may be that other combinations of parameters may be found that give similar degrees of agreement with the observations. For these reasons, other independent observational data – such as the supernova measurements – should also be used to confirm and place further constraints on the values of the parameters, as discussed in Section 7.5.5 of *An Introduction to Galaxies and Cosmology*.

Using a combination of techniques has now allowed the values of the main cosmological parameters to be determined to a high degree of confidence as detailed in Figure 7.27 of *An Introduction to Galaxies and Cosmology*.

Answers to questions

Question 1

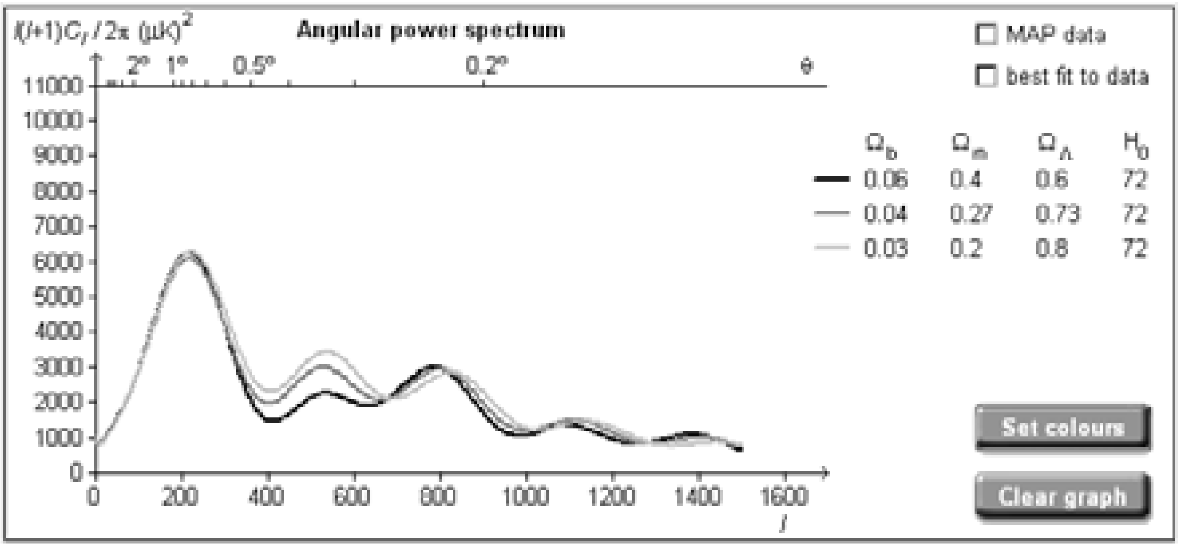
The peaks in the power spectrum at $l = 200$, 500 and 800 correspond to angular scales of about 0.8°, 0.35° and 0.2°, indicating that there is fluctuation on these very small angular scales in the CMB. The COBE map has a much lower angular resolution than this (~7°, see *An Introduction to Galaxies and Cosmology*, Section 7.5.2), so the peaks in this power spectrum would only be seen as fluctuations in the WMAP anisotropy map which has an angular resolution of about 0.1°.

Question 2

Both curves have a peak at around the $l = 200$ mark, so the largest variations in the all-sky maps from these two models would be on similar scales. However, the pattern of the peaks at higher l values is quite different. In particular, there is a large difference between the power spectra at around $l \sim 500$. In the universe that is characterized by the best-fit parameters to the WMAP data, there is a strong peak at $l \sim 500$ and so we would expect to see strong fluctuations on an angular scale of about 0.35°. However, in the critical density model the $l \sim 500$ peak has disappeared, so variations on the corresponding angular scale (0.35°) would be absent from the CMB map in such a universe.

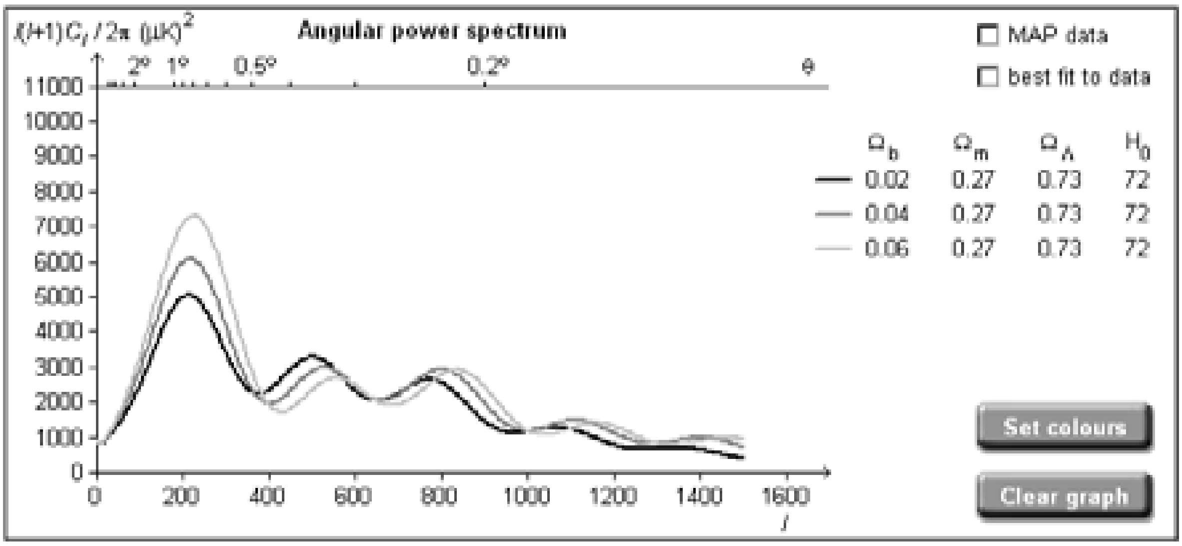
Question 3

You should have obtained the following three plots for the variation of Ω_Λ:



These three curves are all very similar, with peaks in the same locations and only small variations in their amplitudes.

You should have obtained the following three plots for the variation of Ω_b:



These curves show a much larger effect, with both the positions and the amplitudes of the peaks changing. In this comparison, the density of baryonic matter has the greatest effect on the power spectrum of the CMB.

The main process responsible for generating the fluctuations is the interaction of photons with baryonic matter as described in Section 7.5.3 of *An Introduction to Galaxies and Cosmology*. Thus it is hardly surprising that the density parameter of baryonic matter is an important factor in determining the precise shape of the power spectrum.